4. Growth accounting. The Harrod-Domar model.

4.1 Suppose that output, Y, in an economy is produced by combining physical capital, K, with skilled labour, hL, according to a constant-returns *Cobb-Douglas* production function with disembodied technical progress:

$$Y(t) = C(t).K(t)^{0.4}.[h(t).L(t)]^{0.6}$$

where K is the stock of physical capital, L is the labour force and h is the average number of hours *per capita* spent by the labour force on schooling and training. In the last 20 years the labour force grew at an annual rate of 0.6%, the average number of hours of schooling and training per capita of the labour force grew at an annual rate of 1% and the stock of physical capital grew at an annual rate of 2.5%. Assume that the annual growth rate of GDP was 3% in the last 20 years. Calculate the average annual growth rate of the total factor productivity, TFP, in this period.

r(L) = 0.006
r(h) = 0.01
r(K) = 0.025
r(Y) = 0.03
r(C) = ?
r(Y) = r(C) + 0.4r(K) + 0.6r(h) + 0.6r(L)
r(C) = r(Y) - 0.4r(K) - 0.6r(h) - 0.6r(L)
r(C) = 0.03 - 0.4(0.025) – 0.6(0.01) – 0.6(0.006)
≈ 1%

Hence, total factor productivity (TFP = C, in this equation) is a residual (what generally is considered, in statistical analysis, the error term), this is the growth of income that cannot be explained by the growth of factors of production. Some economists call it the "coefficient of ignorance" because it is all the variation of income that is not explained by the variation of inputs, meaning that we do not know much about what it is. Of course, if we add new factors of production to the production function, such as differentiate between skilled and unskilled labour and/or add land, TFP will naturally fall (see, for example, the data at the end of Stephen Broadberry's paper on Kuznets characteristics of modern economic growth). The TFP concept is part and parcel of Solow like models of economic growth.

4.2 Suppose an economy with a Cobb-Douglas aggregate production function with disembodied technical progress, with elasticities of output relative to physical capital equal to 0.3 and to the human capital equal to 0.7. Calculate the average annual growth rate of the labour productivity, assuming that the total factor productivity (TFP) has grown at an annual average rate of 1%, the average number of hours of schooling and training of the labour force has grown at an annual average rate of 0.5% and the stock of physical capital per worker has grown at an annual average rate of 2%.

$Y = C * K^{0.3} * (h.L)^{0.7}$

r(C) = 0.01

r(h) = 0.005

r(K/L) = 0.02

r(Y/L) = ?

$Y = C * K^{0.3} * (h.L)^{0.7}$

$r(Y) = r(C) + 0.3r(K) + [r0.7(h) + 0.7r(L)]$

$\left(\frac{Y}{L}\right) = C * \left(\frac{K}{L}\right)^{0.3} * \left(\frac{h.L}{L}\right)^{0.7}$

$r\left(\frac{Y}{L}\right) = r(Y) - r(L) = r(C) + [0.3r(K) - 0.3r(L)] + 0.7r(h)$

$r\left(\frac{Y}{L}\right) = 0.01 + 0.3(0.02) + 0.7(0.005)$

≈ 1.95%

4.3 The government decided, for the period 2016-2020, the goal of average annual growth rate of labour productivity as 2.5%. The Statistics Office forecasts an average annual growth rate of the labour force of 1.5% for this period, and estimated, also for this period, a capital-output ratio equal to 3 and a depreciation rate of 4%.Assuming the hypotheses of the Harrod-Domar model, make use of it to say what should be, in such conditions, the savings rate of this economy. Make a comment on the hypotheses and on the results.

r(Y/L) = 0.025

r(L) = 0.015

K/Y = 3

δ = 4%

s = ?

To calculate s from H-D basic growth model:

$$\frac{\Delta Y}{Y} = \frac{s}{v} - \delta$$

$$\frac{\Delta Y}{Y} = \frac{s}{3} - 0.04$$

We have two unknows, r(Y) and s, and need r(Y) to calculate s. We can calculate r(Y) from the available information. We know the projected rate of labour productivity, 0.025, and the estimated rate of growth of the labour force, 0.015. Adding both (the additional amount of labour and the average increase in the productivity of all labour, this is, the additional r(Y) that each worker would create), we obtain the projected rate if growth of the Y, r(Y). Such that

r(Y) = r(Y/L) + r(L) = 0.04 (=4%)

$$0.04 = \frac{s}{3} - 0.04$$

$$0.08 = \frac{s}{3}$$

$$s = 0.08 * 3 = 0.24 = 24\%$$

So, s would need to be at 24% (almost a quarter of Y saved for investment, I) in order for r(Y/L) to be at 2.5%, given v, δ and r(L).

4.4 Suppose an economy which functions in accordance with the hypotheses of the Harrod-Domar model and in which total income (Y) increased from 128 billion euros in 2000 to 180 billion euros in 2015. Over the same period, the savings rate was 20% and the depreciation rate was 4%.

a) Estimate the value of the capital stock in 2015.

$Y_{2000} = 128bn$

$Y_{2015} = 180bn$

Number of years, (n) = 15

s = 20% = 0.2

δ = 4% = 0.04

K_{2015} = ?

Capital stock, K, is the amount of capital in existence, which, in this case, given that we know the level of output in 2015 (180bn), is calculated as the amount of capital in existence required to produce that level of output. This is calculated by multiplying the level of output by the capital-output ratio, v. Why? Because the K/L is the "exchange rate" that converts K into Y, this is, the amount of K required to produce one unit of Y. Hence, according to the H-D model, the level of GDP (or Y) in 2015 requires a given stock of physical capital (K) that corresponds to the relationship between K and Y, this is, the amount of K that is required to produce Y, given the "exchange rate" between the two, K/L.

However, v is unknown, but can be calculated from the basic H-D growth equation

$r(Y) = \frac{s}{v} - \delta$ (Harrod-Domar basic growth equation)

$r(Y) = \frac{0.2}{v} - 0.04$, but we cannot solve two unknows, r(Y) and v, with only one equation.

We must solve for r(Y) per year, independently, because v requires r(Y) to be solved.

$$r(Y)_{per\ year} = \left[\frac{ln\left(\frac{Y_{2015}}{Y_{2000}}\right)}{n}\right]$$

$$n = 2015 - 2000 = 15$$

$r(Y)_{per\ year} = \left[\frac{ln\left(\frac{180bn}{128bn}\right)}{15}\right] \approx 0.023$ (=2.3% average growth rate per year)

Attention, the calculation above is of the continuous growth rate. The same calculation can be made using the discrete method. This would be:

$$r(Y)_{per\ year} = \left(\frac{Y_{2015}}{Y_{2000}}\right)^{\left(\frac{1}{n}\right)} - 1 \approx 0.023$$

Now, we can solve for v

$$0.023 = \frac{0.2}{v} - 0.04$$

$$0.063 = \frac{0.2}{v}$$

$$v = \frac{0.2}{0.063} = 3.17$$

Now, we can solve for K_{2015}

$$K_{2015} = v * Y_{2015}$$

$$K = 3.17 * 180bn \approx 571bn$$

So, given v and Y_{2015}, $K_{2015} = 571bn$, or 571bn worth of K were needed to generate 180bn worth of Y, given v = 3.17.

Of course, we know nothing about what K is in real terms (K is an aggregate concept for "physical capital" that is made of many different things, from railways to machines, buildings, roads or vehicles, and K also refers to the role played by those "physical capacities" – for example, a personal vehicle or house for personal consumption differs from a vehicle or building used for production of surplus value). This problem is true for all macroeconomic models that need an aggregate measure of capital. Although in this exercise we are not concerned with this issue, in real economic life this is very important. It also becomes important when we try to answer the questions that follow. Just bear this in mind as part of your education as economists (please, refer to the "capital controversy" referred to in the theoretical classes).

b) If the capital-output ratio had been larger, with all else constant, would the economy have grown faster or slower? Explain why that is the case.

The economy would have grown slower.

$$\frac{\Delta Y}{Y} = \frac{s}{v} - \delta$$

Of course, we can say that dividing a number, numerator, by a larger number, denominator, gives as a smaller result. So, if v is larger, given any level of s, r(Y) is going to be smaller. However, we need to understand the economics of this – why this is so in the H-D model. Is it a freak result, or the result of an inadequate formulation of the model? Or is it what the model, given its own economic logical, is trying to tell us?

$v = \frac{r(K)}{r(Y)}$, is the amount of additional K that is required to produce each additional unit of Y. In other words, v is the converter, or the "exchange rate", of s into r(Y). Hence, a larger v means that a larger amount of K is required for each additional unit of Y, so that for each given value of s, a larger v means a smaller r(Y). Depending on the stock, structure and technology of K, a larger v may (or may not) result in a larger δ (rate of depreciation). If r(Y) slows down, s will slow down in the future, and the combined effect of a smaller "s" and higher "v" is an increasingly slower r(Y).

c) According to the logic of the Harrod-Domar model, what factors influence economic growth and what measures can governments take to promote it?

s = (internal + external) savings. Remember that we are seeking s for investment. So, policies that may affect investment negatively (such as high interest rates, which make K more expensive, cutting welfare state, which may incentivise precautionary savings, lower income and income expectations and reduce consumption, stopping production subsidies or educing real income and consumption) may also affect s negatively. Fiscal policy + industrial policy (direct taxes and the utilization of the additional tax revenue for lowering investment costs and increasing economic expectations, such as, for example, investment in R&D, or in directly productive infrastructures, or reduction of living costs – such of basic goods and of basic services) may increase "s" because they increases business expectations, lower production/investment costs, contribute to increase labour productivity and increase social expectations. Utilisation of internal and external sovereign debt or of foreign direct investment are also options (remember that "s" can include "debt" includes internal and external savings), but these require thinking about and planning for debt sustainability (the level of debt that is consistent with maximizing sustainable economic growth, which is related with the level and the structure of debt, the rate and structure of economic growth, and patterns of income distribution).

$\boldsymbol{v} = \frac{\boldsymbol{r(K)}}{\boldsymbol{r(Y)}}$: the question in how to increase the efficiency of capital, the reciprocal of v, $v^{-1} = \frac{1}{v}$. Industrial policy is one possible tool: skills, technology, social organization of production, management, allocation of investment, efficient scales of production, efficient levels of utilization of capacity, development of production networks, market expansion, etc.

$\boldsymbol{\delta} =$ rate of depreciation of capital (technically given, but related to technology, stock of capital)